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Diffraction of a Plane Wave by a Unidirectionally Conducting Half-plane

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Abstract

Closed form expressions are obtained for the electromagnetic field produced by the diffraction of a plane wave by a semi-infinite unidirectionally conducting screen, and simple far field formulas are given.

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1. Introduction

In a recent paper[1], Toraldo di Francia has given an approximate solution for the diffraction of a plane wave by a small circular disc composed of fine parallel wires. He idealized such a screen as an infinitely thin disc which is perfectly conducting in one direction; the reformulation led to a boundary value problem which was, in turn, solved approximately. During a visit to New York University, the above author suggested to us the problem of determining an exact solution of the correspondingly formulated problem for a half plane. The present paper is devoted to the determination of such an exact solution. The existence of a solution supports the validity of the idealization employed by Toraldo di Francia. It is anticipated that our solution will also prove useful in constructing approximate solutions for the diffraction by large but finite unidirectional screens, just as Sommerfeld's solution for the perfectly conducting half plane proves useful in solving problems of diffraction by large perfectly conducting screens.

In Section 2 we present the mathematical formulation of the boundary value problem. In Section 3 we reduce the problem to the determination of a single scalar wave function. This wave function is then obtained explicitly (Section 4) in terms of functions associated with the diffraction by a perfectly conducting half plane. A simplified far field representation is obtained in Section 5, while Section 6 is devoted to compilation of the results and to brief concluding remarks.

2. Formulation of the boundary value problem

We shall first describe the geometry and coordinate system used here, and then specify the incident field and the boundary conditions.

Our screen is a semi-infinite plane, and we choose a right-handed system of cartesian coordinates (x,y,z) where the y axis coincides with the edge of the screen, and the z axis is perpendicular to the plane of the screen. The screen lies in the part of the x-y plane for which x is positive. In addition to the xyz system we introduce a rotated system of coordinates ξ, η, z, where

(1) $$\xi = x \cos \alpha + y \sin \alpha,$$

(2) $$\eta = -x \sin \alpha + y \cos \alpha.$$

The screen is conducting in the ξ direction only. The incident wave is specified by its magnetic field in the form*

(3) $$\vec{H}_{inc} = \vec{A}\, e^{i\vec{k}\cdot\vec{r}}$$

where $\vec{k}$ is a prescribed propagation vector, whose x,y,z components are k_1, k_2, k_3 respectively, with $k_1^2 + k_2^2 + k_3^2 = k^2$. We assume the number k to have a small positive imaginary part. The components of the vector $\vec{r}$ in the xyz system are x, y, z respectively. The vector $\vec{A}$ is a prescribed constant vector. Of course the vectors $\vec{k}$ and $\vec{A}$ are not independent; from the divergence condition we have

(4) $$\vec{k} \cdot \vec{A} = 0.$$

(The incident electric vector can be determined from the incident magnetic vector via Maxwell's equations.) Our total electric and magnetic fields are denoted by $\vec{E}$, $\vec{H}$ respectively.

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*A factor $e^{-i\omega t}$ is to be understood as multiplying all our fields.

We now turn to the formulation of the boundary conditions which are intended to express the unidirectional conductivity of the screen. According to Toraldo di Francia (private communication) these are expressible as follows:

$$\left.\begin{aligned} &(5) \qquad E_\xi = 0 \\ &(6) \qquad [H_\xi] \equiv H_\xi(x,y,0^+) - H_\xi(x,y,0^-) = 0 \\ &(7) \qquad [E_\eta] = 0\,. \end{aligned}\right\} \qquad z = 0, x > 0$$

Here the notation E_ξ means the ξ component of the total vector E. That these conditions are reasonable is suggested by their successful employment in the theory of a traveling wave tube constructed of a tightly wound helical wire. (See [2], for example).

In addition to the above conditions Toraldo proposes the edge condition

$$(8) \qquad [H_\eta] \to 0$$

at the edge of the screen. This condition, which is necessary in order to have a unique solution, represents the vanishing of the currents at the ends of the wires composing the screen.

To complete the specification of the fields to be determined, we have to impose the conditions that $(\vec{E} - \vec{E}_{inc})$ and $(\vec{H} - \vec{H}_{inc})$ are to be outgoing at infinity. In view of our use of a complex k, it follows that these vectors vanish at infinity.

In addition to the above conditions we have found it necessary, in analogy with the theory of perfectly conducting screens, to require (cf. Meixner [2]) that our fields are less singular at the edge than $\frac{1}{\rho}$, where $\rho = \sqrt{x^2 + z^2}$.

Of course, in the exterior of the screen, $\vec{E}$ and $\vec{H}$ must satisfy Maxwell's equations

$$(9) \qquad \text{a)}\ \nabla \times \vec{E} = \frac{i\omega}{c}\vec{H} \qquad\qquad \text{c)}\ \operatorname{div}\vec{E} = 0$$

$$\text{b)}\ \nabla \times \vec{H} = -\frac{i\omega}{c}\vec{E} \qquad\qquad \text{d)}\ \operatorname{div}\vec{H} = 0.$$

3. Reduction to a scalar problem

We now reduce our problem to a scalar problem. To this end we note that, from 9a)

$$(10) \qquad \frac{\partial H_\xi}{\partial z} = \frac{c}{i\omega}\frac{\partial}{\partial z}\left[\frac{\partial E_z}{\partial \eta} - \frac{\partial E_\eta}{\partial z}\right],$$

while we know that

$$-\frac{\partial^2 E_\eta}{\partial z^2} = \frac{\partial^2 E_\eta}{\partial \xi^2} + \frac{\partial^2 E_\eta}{\partial \eta^2} + k^2 E_\eta .$$

Using 9c) we infer that

$$(11) \qquad \left[\frac{\partial H_\xi}{\partial z}\right] = \frac{c}{i\omega}\left[-\frac{\partial}{\partial \eta}\left(\frac{\partial E_\xi}{\partial \xi} + \frac{\partial E_\eta}{\partial \eta}\right) + \frac{\partial^2 E_\eta}{\partial \eta^2} + \frac{\partial^2 E_\eta}{\partial \xi^2} + k^2 E_\eta\right].$$

Since $\left[E_\xi\right] = 0$ by (5) and $\left[E_\eta\right] = 0$ by (7), this must also be true of their derivatives in directions tangential to the screen, whence we infer that

$$(11a) \qquad \left[\frac{\partial H_\xi}{\partial z}\right] = 0.$$

We recall, in addition, that $\left[H_\xi\right] = 0$, (cf. equation (6)).

By the above arguments we have established that the wave function H_ξ is continuous together with its derivatives through the screen. It follows that it is a regular wave function everywhere except possibly in the neighborhood of the edge. The same is therefore true of $h_\xi = H_\xi - H_\xi^{\text{inc}}$. The function h_ξ is also outgoing at infinity. Such a function can be developed, by separation of variables, in the form* $e^{ik_2 y}$ multiplied by a series of Hankel functions of integer and zero order. Our restriction on the order of magnitude of edge singularity excludes the non-zero orders. Therefore h_ξ must be either a multiple of $e^{ik_2 y} H_0^{(1)}(K\rho)$ or it must vanish. We adopt the latter ansatz, in order to avoid the possibility of too large $\vec{E}$ fields which would otherwise be generated by equation (9b). We shall show that a solution may be constructed on the assumption $h_\xi = 0$ which will thus be justified. It is to be noted that the simpler problem of reflection and transmission of plane waves by a fully infinite unidirectional screen can be shown to have this behavior.

Since we have $h_\xi = 0$, we deduce from (9d) that

$$\frac{\partial h_\eta}{\partial \eta} + \frac{\partial h_z}{\partial z} = 0, \tag{12}$$

so that we can introduce a scalar wave function u(x,y,z) such that

$$h_\eta = \frac{\partial u}{\partial z} \tag{13a}$$

$$h_z = -\frac{\partial u}{\partial \eta}. \tag{13b}$$

We can then express the quantity

$$\vec{e} = \vec{E} - \vec{E}^{inc}$$

in terms of u via (9b), (13a), and (13b). We find

$$e_\xi = -\frac{c}{i\omega}\left[-\frac{\partial^2 u}{\partial \eta^2} - \frac{\partial^2 u}{\partial z^2}\right] = -\frac{c}{i\omega}\left[\frac{\partial^2 u}{\partial \xi^2} + k^2 u\right] \tag{14}$$

$$e_\eta = -\frac{c}{i\omega}\left[\frac{\partial^2 u}{\partial \xi \partial \eta}\right]$$

$$e_z = -\frac{c}{i\omega}\left[\frac{\partial^2 u}{\partial \xi \partial z}\right].$$

---

*This form is valid for a straight edge, but the ensuing result may be plausibly extended to any smooth edged screen, since it is a statement about behavior in the small ($\rho \to 0$).

Since our incident fields are certainly regular at the screen, the conditions (5), (6), (7), and (8), become

$$\left.\begin{array}{ll}(15) & \dfrac{\partial^2 u}{\partial \xi^2} + k^2 u = -\dfrac{i\omega}{c} E_\xi^{inc} \\[2ex] (15a) & \left\{\dfrac{\partial^2}{\partial \xi^2} + k^2\right\} [u] = 0; \text{ i.e., } [u] = A(\eta)e^{ik\xi} + B(\eta)e^{-ik\xi}\end{array}\right\} \text{for } z = 0,\ x > 0$$

$$\left.\begin{array}{ll}(16) & \dfrac{\partial^2}{\partial\xi\partial\eta}[u] = 0 \\[2ex] (17) & \left[\dfrac{\partial u}{\partial z}\right] = 0 \text{ at the edge}\end{array}\right\} \text{for } z = 0 \text{ and } x > 0 .$$

The natural procedure is to obtain the general solution for the values of u on the screen from (15), in the form of a particular solution plus an undetermined complementary function. Using these values of u as boundary conditions, we determine the outgoing wave function u in all space, and compute $\left[\frac{\partial u}{\partial z}\right]$. Then we impose condition (17) and thereby determine the complementary function, whereupon u is determined. This procedure will be followed in the ensuing section.

4. Determination of function u(x,y,z)

The concluding remarks of Section 3 constitute a program which is applicable to any screen whatever, but of course we have not shown that a suitable complementary function, allowing the fulfillment of (17), always exists. We shall now call in to play the fact that we are dealing with a half-plane in order to simplify the matter further. To this end we assume a special form of solution, based on the fact that the conditions of the problem depend on y only via the incident wave. Accordingly, we assume that (as is the case for the incident wave), all our field quantities have the form $e^{ik_2 y}$ times a function of x and z alone. Thus we set $u = \hat{u}(x,z)e^{ik_2y}$. Since all our fields are wave functions, this implies that

$$(18)\qquad \frac{\partial^2\hat{u}}{\partial x^2} + \frac{\partial^2\hat{u}}{\partial z^2} + (k^2 - k_2^2)\,\hat{u} = 0.$$

Now from (15a) we have

$$(19)\qquad [u] = A(\eta)e^{ik\xi} + B(\eta)e^{-ik\xi}.$$

Differentiating (19) with respect to ξ and η, and applying (16), we conclude that

$$(20)\qquad A'(\eta) \equiv B'(\eta) \equiv 0,$$

in view of the absurd result ($e^{-2ik\xi}$ = constant) which follows from the contrary hypothesis.

Now we note that if $\vec{k}$ is in the ξ direction, we have $E_\xi^{inc} \equiv 0$ (since $\vec{E}^{inc} \cdot \vec{k} = 0$), and then the solution of our problem is, trivially, $\vec{E} = \vec{E}^{inc}$, $\vec{H} = \vec{H}^{inc}$. We exclude this case from what follows. Then, from (15), we have

$$(21)\qquad u_{\xi\xi} + k^2u = De^{i(k_1x + k_2y)};\quad D = -\frac{i\omega}{c}E_\xi^{inc}(0,0,0).$$

The general solution is

$$(22)\qquad u(x,\pm 0,y) = A_1^{\pm}(\eta)\,e^{ik\xi} + B_1^{\pm}(\eta)\,e^{-ik\xi} + D_1e^{i(k_1x + k_2y)}$$

where the superscripts denote the value on upper and lower sides of the screen, and where

$$(22a)\qquad D_1 = D/(k^2 - (k_1\cos\alpha + k_2\sin\alpha)^2) = -\frac{i\omega}{c}E_\xi^{inc}(0,0,0)/\left[k^2 - (k_1\cos\alpha + k_2\sin\alpha)^2\right].$$

Now for a half plane as noted above, we have $u = \hat{u}(x,z)e^{ik_2y}$ so that we must have

$$(23) \qquad \hat{u}(x,\pm 0) = A_1^{\pm}(\eta)\, e^{i(k\xi - k_2y)} + B_1^{\pm}(\eta)\, e^{-i(k\xi + k_2y)} + D_1 e^{ik_1x},$$

where the right side of (23) must be independent of y. We differentiate with respect to y and set the result equal to 0, and obtain

$$(24) \quad \left[A_1'(\eta)\cos\alpha + i\, A_1(k\sin\alpha - k_2)\right]e^{ik\xi} + \left[B_1'(\eta)\cos\alpha - iB_1(k\sin\alpha + k_2)\right]e^{-ik\xi} = 0$$

which implies that the coefficients of $e^{ik\xi}$ and $e^{-ik\xi}$ must vanish. It follows from (24) that

$$(25) \qquad A_1^{\pm}(\eta) = a_1^{\pm}\exp\left[i(-k\tan\alpha + k_2\sec\alpha)\eta\right]$$

$$B_1^{\pm}(\eta) = b_1^{\pm}\exp\left[i(k\tan\alpha + k_2\sec\alpha)\eta\right],$$

where $a_1^{\pm}$ and $b_1^{\pm}$ are constants. Substituting in the expression (23) for $\hat{u}(x,0)$ we find

$$(26) \qquad \hat{u}(x,\pm 0) = a_1^{\pm}\, e^{ic_1x} + b_1^{\pm}\, e^{ic_2x} + D_1 e^{ik_1x}; \qquad x > 0$$

where

$$(27) \qquad c_1 = k\sec\alpha - k_2\tan\alpha$$

$$(28) \qquad c_2 = -k\sec\alpha - k_2\tan\alpha.$$

Now our incident wave has a real angle of incidence, whence it follows that $k_2 = k \cdot \gamma$ where the direction-cosine γ is less than 1 in absolute value. Since k has a positive imaginary part, we can now infer that c_1 has a positive imaginary part while c_2 has a negative imaginary part. The wave function u should be finite at infinity, and consequently we must discard the term in c_2

which is exponentially large at $x = +\infty$.

From the resulting simplification of (26) and from (20), we can now conclude that $[u] = 0$, unless $k \sin \alpha = k_2$. We shall postulate that $[u] = 0$ in any case.* The E and H field so obtained is verifiable by direct calculation.

In view of equations (17), (18) and (26), and in view of the arguments of the last paragraph, our boundary value problem for $\hat{u}(x,z) \equiv u(x,y,z)e^{-ik_2y}$ can therefore be expressed as follows:

$$(29) \qquad \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + K^2\right)\hat{u} = 0; \qquad K^2 = k^2 - k_2^2 > 0 .$$

$$(30) \qquad \hat{u} \to 0 \text{ at } \infty.$$

$$(31) \qquad \hat{u}(x,0) = a_1e^{ic_1x} + D_1e^{ik_1x}; \qquad x > 0$$

$$(32) \qquad \left[\frac{\partial\hat{u}}{\partial z}\right] \to 0 \text{ as } x \to 0^+ .$$

These equations suggest that we solve for $\hat{u}$ in the form

$$(33) \qquad \hat{u}(x,z) = a_1v(x,z) + D_1w(x,z)$$

where

$$(34) \qquad v = e^{ic_1x}; \quad w = e^{ik_1x} \qquad \text{for } z = 0,\ x > 0,$$

and both v and w vanish at ∞. We shall see that a solution can actually be achieved by this ansatz.

After solving for v and w, we must compute $[v_z]$ and $[w_z]$, and choose the constant a_1 so as to enforce the condition $[u_z] = 0$ at the edge. Now it is apparent that the function w is nothing but the scattered field produced by the diffraction of the plane wave $\exp\left[i(k_1x + \sqrt{K^2-k_1^2}z)\right]$ at the half plane $z = 0$, $x > 0$. The solution of this two-dimensional problem is well known. On the other hand, the function v cannot be interpreted in this way, since we cannot form a two-dimensional plane wave* (with real angle of incidence) which adopts the value e^{ic_1x} at $z = 0$. It is however possible to solve for v by a simple modification of a method due to Lamb[4]. For the sake of completeness we also give the determination of the function w by this method.

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*Actually, if $k \sin \alpha = k_2$ the foregoing analysis merely shows that $[\hat{u}] = [a_1]\exp(ic_1x) = [a_1]\exp(ik \cos \alpha x)$. But, at a later stage of the argument, after applying the edge condition on $[H_n]$ one could show the necessity of $[a_1] = 0$, i.e. of $[u] = 0$.

Let us first determine v. We define a function $\Psi(x,z)$ by the equation

$$(35) \qquad \Psi(x,z) = \frac{\partial v}{\partial x} - ic_1v = e^{ic_1x}\frac{\partial}{\partial x}(ve^{-ic_1\Psi}).$$

The function Ψ must vanish at $z = 0$, $x > 0$. Let $\rho = \sqrt{x^2+z^2}$, $\theta = \arctan\frac{z}{x}$, $0 \leq \theta \leq 2\pi$. Then a solution Ψ obeying the boundary conditions is

$$(36) \qquad \Psi = \Psi_0 \frac{e^{iK\rho}}{\sqrt{\rho}} \cdot \sin\theta/2 \; ; \qquad \Psi_0 = \text{constant.}$$

We now solve the ordinary differential equations (35) for v and obtain

$$(37) \qquad v(x,z) = e^{ic_1x}\left[\int_{x_0=-\infty}^{x} \exp[-ic_1x_0]\,\Psi(x_0,z)\,dx_0 + f(z)\right].$$

Here f(z) is a constant of integration. Using the fact that Ψ is a wave function in order to calculate

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*Note $K^2 - c_1^2 = -k^2(\sin\alpha - \sin\beta)^2 < 0$, if we define $\sin\beta = k_2/k$.

$$\int_{-\infty}^{x} \exp\left[-ic_1x_0\right] \Psi_{zz} dx_0 = -\int_{-\infty}^{x} \exp\left[-ic_1x_0\right](\Psi_{x_0x_0} + k^2\Psi)dx_0,$$

by partial integration, we find that the first term in our expression for v is a wave function. Thus the function f(z) must be of the form

$$f_1 \exp\left[iz\sqrt{K^2 - c_1^2}\right] + f_2 \exp\left[-iz\sqrt{K^2 - c_1^2}\right],$$

(where f_1 and f_2 are constants), if the whole expression (37) for v is to be a wave function. However, one of these terms is exponentially large as $z \to +\infty$, and the other is large as $z \to -\infty$, provided $k_2 \neq k \sin \alpha$. (If $k_2 = k \sin \alpha$, then the terms are constants). On the other hand, the integral term occurring in (37) goes to zero as $|z| \to \infty$, as can be seen by substituting the formula for Ψ in the integral. Since we require that $v \to 0$ at infinity, the above remarks show that $f_1 = f_2 = 0$ (whether or not $k_2 = k \sin \alpha$).

We have yet to determine Ψ_0. This can be done by evaluating v(x,0) for x positive, from equation (37), and setting it equal to e^{ic_1x}. To do so we first note that for $z = 0$, $x_0 > 0$, $\Psi(x_0,0) = 0$, while for $z = 0$, $x_0 < 0$, we have

$$(38) \qquad \Psi(x_0,0) = \Psi_0 \frac{\exp\left[iK|x_0|\right]}{\sqrt{|x_0|}},$$

since $\Theta = \pi/2$ in this range. Consequently we obtain

$$v(x,0) = \Psi_0 e^{ic_1x} \int_{x_0=-\infty}^{0} e^{-ic_1x_0} \frac{\exp\left[iK|x_0|\right]}{\sqrt{|x_0|}} dx_0 = e^{ic_1x}$$

whence it follows that

(39) $$\Psi_o = 1 \Big/ \int_{-\infty}^{o} \frac{\exp\left[-i(K+c_1)x_o\right]}{\sqrt{-x_o}}\, dx_o = \sqrt{\frac{K+c_1}{\pi}}\; e^{-i\pi/4} ,$$

and

(39a) $$v(x,z) = \exp\left[ic_1x\left[\int_{x_o=-\infty}^{x} \exp(-ic_1x_o)\Psi(x_o,z)dx_o\right]\right] .$$

We determine w similarly, by introducing the function

$$\phi = \frac{\partial w}{\partial x} - ik_1w.$$

The function w must represent outgoing waves at ∞. Of course, as is well known, in certain regions of space, these outgoing waves will be partly plane waves. In fact, w is the negative of that scattered wave which corresponds to the incident wave $\exp\left[ik_1x + iz\sqrt{K^2 - k_1^2}\,\right]$, and therefore w will contain shadow-forming waves or reflected waves, in the appropriate regions of space indicated by geometrical optics. However, such waves[*] depend on x in the form e^{ik_1x}. Consequently $\phi = w_x - ik_1w \to 0$ at ∞. Thus

(40) $$\phi = \phi_o \frac{e^{iK\rho}}{\sqrt{\rho}} \sin\frac{\theta}{2} , \quad \phi_o = \text{constant}.$$

Solving for w we obtain

(41) $$w = e^{ik_1x} \int_{-\infty}^{x} \exp\left[-ik_1x_o\right] \phi(x_o,z)dx_o$$

where the homogeneous exponential solutions which are otherwise possible are rejected as violating the outgoing condition. The constant ϕ_0 is determined by the same procedure that we employed in the case of the function v. We find

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[*] The shadow-forming wave is the negative of the incident wave while the reflected wave has the form $\exp(ik_1x - iz\sqrt{K^2-k_1^2}\,)$. Thus both are proportional to $\exp(ik_1x)$, as remarked above.

$$(42) \qquad \phi_0 = 1 \Big/ \int_{-\infty}^{0} e^{-ik_1x_0} \exp\left[\frac{iK|x_0|}{\sqrt{|x_0|}}\right] dx_0 = \sqrt{\frac{K+k_1}{\pi}}\; e^{-i\pi/4} .$$

We have now determined the functions v and w, and we have

$$\hat{u}(x,0) = a_1 v + D_1 w.$$

The constant a_1 is now to be determined by calculating $\left[\frac{\partial \hat{u}}{\partial z}\right]$ and requiring it to vanish as $x \to 0^+$.

In order to study the behavior of $\left[\frac{\partial \hat{u}}{\partial z}\right]$ it is useful to notice that the wave functions $(v - \exp\left[i(c_1x + \sqrt{K^2-c_1^2}\; z)\right])$ and $(w - \exp\left[i(k_1x + \sqrt{K^2-k_1^2}\; z)\right])$ vanish on the screen, and are finite at the edge. Consequently they can be developed near the edge in Fourier series of the form

$$\sum_{n=1}^{\infty} c_n J_{n/2} (K\rho) \sin \frac{n}{2}\, \theta.$$

In these series the terms of odd index have discontinous z derivatives at the screen. The leading terms are $O(\sqrt{r} \sin \theta/2)$ and give rise to a discontinuity in $\frac{\partial v}{\partial z}$ and $\frac{\partial w}{\partial z}$ of order of magnitude $r^{-1/2}$. The remaining terms of $\left[\frac{\partial u}{\partial z}\right]$ tend to zero. Therefore we must choose the constant a_1 so as to cause the leading terms of $a_1 \frac{\partial v}{\partial z}$ and $D_1 \frac{\partial w}{\partial z}$ to cancel each other. This is achieved by selecting

$$(43) \qquad a_1 = - \frac{D_1\phi_0}{\Psi_0} = - D_1\sqrt{\frac{K+k_1}{K+c_1}}$$

as is evident from the fact that

$$a_1\Psi + D_1\phi = a_1\left(\frac{\partial}{\partial x} - ic_1\right) v + D_1\left(\frac{\partial}{\partial x} - ik_1\right) w$$

then vanishes, whence $a_1 \frac{\partial v}{\partial x} + D_1 \frac{\partial w}{\partial x}$ must be finite, whence in turn the odd index terms in $a_1 v + D_1 w$ must vanish more rapidly than $\sqrt{r} \sin \theta/2$.

We have now determined the constant a_1 and the functions v and w. (These functions can be expressed as Fresnel integrals, but this is not necessary for our purposes.) We now observe that via equations (40) to (42), (39) and (39a), and (22a), and (33) and (43) we have a determination of $\hat{u}(x,z)$, and of $u(x,y,z) \equiv \hat{u}(x,z)\, e^{ik_2 y}$. We then obtain h_η and h_z from equations (13) and (13a). Now, using (9b) we can determine the scattered field $\vec{e}$, and our solution is complete. This solution is collected and given explicitly in Section 6. Inspection shows that it is valid whether or not $k \sin \alpha = k_2$. Thus that restriction is irrelevant, as noted earlier.

5. Determination of the far field

It is of interest to supply a determination of the far field. In order to do this we note that, since $u(x,y,z) \equiv \hat{u}(x,z)e^{ik_2 y}$, we expect a representation of the far field to have the standard form,

$$(44) \qquad u \sim e^{ik_2 y} \left\{ \frac{e^{iK\rho}}{\sqrt{\rho}} U(\theta) + D_1 \exp\left[ik_1 x + ik_3 |z|\right] m(\theta) \right\} , \qquad \rho \to \infty,$$

where $m(\theta)$ is a step function (whose form we indicate subsequently) which represents the shadow-forming and reflecting properties of the screen. Our object is now to determine $U(\theta)$. Once $U(\theta)$ is determined, we can obtain the corresponding representations of the cartesian components of $\vec{e}$ and $\vec{h}$. Now we note that

$$(45) \qquad u = a_1 v + D_1 w,$$

so that to obtain $U(\theta)$ we only need to know the amplitude functions $V(\theta)$, $W(\theta)$

such that

$$(46) \qquad \left.\begin{aligned} v &\to \frac{e^{iK\rho}}{\sqrt{\rho}} V(\theta) \\ w &\to \frac{e^{iK\rho}}{\sqrt{\rho}} W(\theta) + m(\theta) \exp\left[ik_1x + ik_3|x|\right] \end{aligned}\right\} \text{ as } \rho \to \infty \text{ with fixed } \theta.$$

We now must specify the function $m(\theta)$, and then determine $V(\theta)$ and $W(\theta)$. The function $m(\theta)$ is readily obtained from geometric considerations since the term it affects represents the value of w according to geometrical optics. We have

$$(48) \qquad m(\theta) = 1, \qquad 0 < \theta \arctan \frac{k_3}{k_1} = \arctan\left\{ \frac{\sqrt{K^2=k_1^2}}{k_1} \right\} \equiv \theta_o ,$$

(the shadow region),

$$m(\theta) = 0, \qquad \theta_o < \theta < 2\pi - \theta_o$$

$$m(\theta) = 1, \qquad 2\pi - \theta_o < \theta < 2\pi. \qquad \text{(the reflection region).}$$

The lines $\theta = \theta_o$, $\theta = 2\pi - \theta_o$ are called 'shadow lines' in the literature. On these lines the function $w(\theta)$ becomes infinite, as we shall see, (cf. (51)), and the asymptotic representation (46) fails.

These results follow from the fact that at infinity, in the successive angular regions indicated in (48), w must successively cancel the incident wave to form a shadow, leave the incident wave unperturbed, or provide a reflected wave. The functions $V(\theta)$, $W(\theta)$ are now obtained as follows. We have

$$(49) \qquad \frac{\partial v}{\partial x} - ic_1v = \phi_o \frac{e^{iK\rho}}{\sqrt{\rho}} \sin \theta/2$$

$$\frac{\partial w}{\partial x} - ik_1w = \Psi_o \frac{e^{iK\rho}}{\sqrt{\rho}} \sin \theta/2.$$

We express the differentiations in polar coordinates, apply them to the asymptotic representations given in equation (46), and reject terms of higher order than $\rho^{-1/2}$. Then we obtain

$$i(K \cos \Theta - c_1)\, V(\Theta) = \Psi_o \sin \frac{\Theta}{2}$$

$$i(K \cos \Theta - k_1)\, W(\Theta) = \phi_o \sin \frac{\Theta}{2} .$$

Consequently

$$V(\Theta) = \frac{\Psi_o \sin \Theta/2}{i(K \cos \Theta - c_1)} \tag{50}$$

$$W(\Theta) = \frac{\phi_o \sin \Theta/2}{i(K \cos \Theta - k_1)} . \tag{51}$$

From equation (45) we deduce that $U(\Theta) = a_1 V(\Theta) + D_1 W(\Theta)$, whence

$$U(\Theta) = \frac{\sin \Theta/2}{i} \left\{ \frac{a_1 \Psi_o}{K \cos \Theta - c_1} + \frac{D_1 \phi_o}{K \cos \Theta - k_1} \right\} \tag{52*}$$

for any angle Θ which is bounded away from the angles Θ_o, $2\pi - \Theta_o$, where $\Theta_o = \arctan k_3/k_1$. The far field form of h_η and h_ξ can be obtained from the above equations by the asymptotic differentiation process already employed. The relevant formulas for differentiating the terms involving $u(\Theta)$ and $v(\Theta)$ are

$$\frac{\partial}{\partial z} \doteq iK \sin \Theta ; \ \frac{\partial}{\partial y} \doteq ik_2 ; \ \frac{\partial}{\partial x} \doteq iK \cos \Theta ,$$

so that

$$h_z = - \frac{\partial u}{\partial \eta} \cong - \frac{\exp\left[ik_2 y + iK\rho\right]}{\sqrt{\rho}} \cdot (-\sin \alpha \cdot iK \cos \Theta + \cos \alpha \cdot ik_2)\, U(\Theta) \tag{53}$$

$$- D_1\, m(\Theta) \left\{-ik_1 \sin \alpha + ik_2 \cos \alpha\right\} \exp\left[ik_1 x + ik_3 |z|\right] ,$$

---

*Note that we can never have $K \cos \Theta = c_1$ for a real non-zero Θ, in view of the footnote on Page 9. Thus, the only shadow lines are those of w, as indicated in (48).

$$(54)\quad h_\eta = \frac{\partial u}{\partial z} \cong \frac{\exp\left[ik_2y + iK\rho\right]}{\sqrt{\rho}} \left\{ iK \sin\theta\, U(\theta) \right\} + D_1 m(\theta) ik_3 \frac{z}{|z|} \exp\left[ik_1x + k_3|z|\right].$$

These are the results we aimed at in this section.

6. Conclusion

A solution has been obtained for the diffraction of the electromagnetic planewave

$$\vec{H}^{inc} = \vec{A}\, e^{i\vec{k}\cdot r} = \vec{A} \exp\left[i(k_1x + k_2y + k_2z)\right]$$

$$\vec{E}^{inc} = -\frac{c}{i\omega} \nabla \times \vec{H}^{inc},$$

by a screen which occupies the semi-infinite plane $x > 0$, $z = 0$, and which is perfectly conducting in the direction of the ξ axis, where

$$\xi = x\cos\alpha + y\sin\alpha$$

$$\eta = -x\sin\alpha + y\cos\alpha,$$

α being a prescribed angle, $0 \leq \alpha < \frac{\pi}{2}$, under the restriction that $k_2 - k\sin\alpha$ and k_3 are not simultaneously zero. The field obtained is a solution of the boundary value problem formulated in Section 2, pp. 2, 3. We conclude that the idealization embodied in this boundary value problem makes sense. The solution we obtained is defined by

$$\vec{H}_{total} = \vec{H}_{inc} + \vec{h}$$

$$\vec{E}_{total} = -\frac{c}{i\omega} \nabla \times \vec{H}$$

where

$$\vec{h} = (h_\xi, h_\eta, h_z)$$

with

$$h_\xi = 0;\ h_\eta = \frac{\partial u}{\partial z};\quad h_z = -\frac{\partial u}{\partial \eta}\ .$$

For u we have the formula

$$u \equiv \hat{u}(x,z)\, e^{ik_2 y}$$

$$= e^{ik_2 y} \left[D_1\right] \left[-\phi_0\, e^{ic_1 x} \int_{-\infty}^{x} e^{-ic_1 x_0} \frac{e^{iK\rho}}{\sqrt{\rho}} \sin \theta/2 \, dx_0 \right.$$

$$\left. + \phi_0\, e^{ik_1 x} \int_{-\infty}^{x} e^{-ik_1 x_0} \frac{e^{iK\rho}}{\sqrt{\rho}} \sin \theta/2 \, dx_0 \right]$$

where

$$c_1 \equiv k \sec \alpha - k_2 \tan \alpha$$

$$D_1 \equiv -\frac{i\omega}{c} E_\xi^{inc} \Big/ \left[k^2 - (k_1 \cos \alpha + k_2 \sin \alpha)^2\right]$$

$$\phi_0 \equiv \sqrt{\frac{K+k_1}{\pi}}\, e^{-i\pi/4} .$$

Simple asymptotic representations of the solution which are valid for large ρ are indicated in Section 5. Note that if $\vec{H}^{inc}$ is in the ξ direction then $E_\xi^{inc} = 0$, whence $D_1 = 0$, $u \equiv 0$, and $\vec{h} \equiv 0$, so that the screen is transparent to a plane wave whose incident magnetic vector is parallel to the direction of conductivity.

In our analysis we have excluded the case where $k \cos \alpha = k_1$, $k \sin \alpha = k_2$, $k_3 = 0$, i.e. $\vec{k}$ in the direction of the ξ axis. However, in this excluded case we have $\vec{E}_\xi^{inc} = 0$ and $H_\xi^{inc} = 0$, since the incident E and H vectors are orthogonal to the direction of propagation. Then we again have the trivial solution $\vec{E} = \vec{E}^{inc}$, $H = \vec{H}^{inc}$, since the boundary conditions are then trivially fulfilled.

Our solution is therefore complete.

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